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rate topics carefully partitioned off from each other both theoretically and pedagogically.

The editors of the MONTHLY have decided to open its columns to contributions on the teaching of collegiate mathematics, in the hope that some impetus may be given to the better training of teachers, the better arrangement and coordination of material and the better form and methods of presentation. Already some important papers have been promised and others will be announced in the near future.

Moreover, no less than six sub-committees, under Klein's International Commission on the Teaching of Mathematics, are now working in this country on topics connected with mathematics of a collegiate grade, and it is supposed that the substance of their reports will be made known in America and will become the basis of discussion at the various meetings of mathematical societies during the Autumn and Winter. From these sources also much should be expected both in pointing out existing conditions and in arousing interest in questions of possible improvements.

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## ON A FEW POINTS IN THE HISTORY OF ELEMENTARY MATHEMATICS.

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The main object of the present note is to call attention to the recent changes of view in reference to several important questions in the history of elementary mathematics. Among the general works on the history of mathematics there is probably none which is more commonly trusted than Cantor's *Vorlesungen über Geschichte der Mathematik*. The first volume of this work is now in its third edition, the second and third volumes are in second editions, while the first edition of the fourth volume appeared as recently as 1908. As the first edition of the first volume appeared in 1880, the rapid succession of new editions of such an extensive work is an index of the rapid recent progress in the development of the history of mathematics. Discoveries have followed each other in such close succession as to call for rapid changes of view even in regard to some of the most fundamental matters.

In support of this statement we shall give a few changes of view as exhibited in the second and third editions of volume I of Cantor's monumental work. On page 576 of the second edition the following words may be found, "According to our opinion the discovery of zero is due to the Hindus." The corresponding statement in the third edition, page 616, reads as follows: "According to our opinion the discovery of zero is due to the Baby-

lonians, the deepening of the concept is due to the Hindus." It need scarcely be added that the former of these two views is expressed in nearly all works of reference relating to this subject, but it is likely that this will gradually be changed on account of the great influence of Cantor in historical matters and the strong reasons advanced by him in support of his change of view.

The discovery of zero, as used above, implies its use in positional arithmetic. It is certain that the Greeks employed zero in the second century B. C. to denote the absence of degrees, minutes, or seconds in their sexagesimal notation.\* The earliest known use of this symbol in Babylonian inscriptions belongs to the third century B. C., but it is supposed that it was in use at a much earlier date. At the international mathematical congress held in Paris in 1900 Cantor suggested that zero was probably in use among the Babylonians as early as 1700 B. C. Even if such an early date cannot be established it appears likely that scholars will hereafter attribute the discovery of positional arithmetic to the Babylonians instead of to the Hindus.

Another important change of view exhibited in the two editions mentioned above relates to the sexagesimal system of notation, which is still used by us in measuring time and angles. In the second edition, page 92, Cantor says that the ancient Babylonian astronomers probably thought the year was composed of 360 days and hence they divided the circle into 360 parts, each part corresponding to a day of the year. An apparent corroboration of this view was furnished by an ancient Chinese custom to divide the circle into  $365\frac{1}{4}$  degrees. After the circle was divided into  $360^\circ$ , it is easy to see that the base 60, for the sexagesimal system, might have been suggested by the fact that the side of a regular inscribed hexagon is equal to the radius of the circle.

In the third edition Cantor abandons this view and suggests that the base 60 may have resulted from the union of two nations, meeting in Babylon, one having a system of numeration with 10 as a base while the other employed 60 as a base. Among the reasons advanced for abandoning his former position are the following: The counting and writing of small numbers must have been known long before such comparatively large numbers as 360 were in use, and it must have been recognized by the early Babylonian astronomers that the year involves more than 360 days. Hence it does not appear likely that there is any connection between the number of degrees in a circle and the supposed number of days in the year.

These changes of view should influence the high school teacher to supplement the historical notes even in many of the best and most recent text-books. Whenever the teacher expresses an opinion in reference to a question which has not been fully settled this opinion should be in accord with the most advanced scholarship. The changes of view of such an eminent scholar as Moritz Cantor should also impress the teacher with the fact

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\**Encyclopédie des Sciences Mathématiques*, Vol. 1, page 17.

that the most modern works should be consulted when one is dealing with historical data. What the foremost scholar along a certain line endorsed ten years ago may be antiquated rubbish to-day.

A less positive but scarcely less important change of view, exhibited in the two editions under consideration, relates to the development of Egyptian mathematics at the time of Ahmes, about 1700 B. C. As the extraction of the square root does not occur in the work of Ahmes, it was natural to infer that the Egyptians were unacquainted with this operation at this early date. In the third edition we are told, on the contrary, that the ancient Egyptians knew how to extract the square root and even how to solve such systems of simultaneous equations as

$$\begin{aligned}x^2 + y^2 &= 100 \\ x : y &= 1 : \frac{3}{4}.\end{aligned}$$

These recent discoveries prove that the ancient Egyptians knew much more about equations than what was previously supposed.

Recent discoveries in Babylon have also led to changes of view. Many of the histories of mathematics state that the Babylonians did not use any number as large as a million and some historians have devoted considerable attention to this supposed fact. The recent discoveries by Hilprecht of the University of Pennsylvania have revealed that the Babylonians made use of as large numbers as

$$195, 955, 500, 000, 000$$

and that numbers exceeding a million were frequently employed.\* Even in the latest edition of Cantor's *Vorlesungen* the obsolete view is expressed.

In closing this note it may be in place to quote the remarks of J. J. Milne before a meeting of British teachers of mathematics, as follows: "Speaking as a schoolmaster to schoolmasters I think we ought to bring the history of mathematics more than we do before the notice of our pupils. Mathematics is a living, growing science, with a definite history, and there is not a branch of it which boys take up in school, whether arithmetic or algebra, or geometry, or trigonometry, or any other of the many divisions, but has its own history, and I have always found that boys are interested in learning what properties were known to the ancients, and what have been discovered in modern times, and I often think that the writers of text-books would do well to devote a little more space than they do to what I may call the note of human interest." For instance, in teaching complex fractions it is of interest to observe that the Hindus of the ninth century taught the rule of inverted divisor and that this rule was rediscovered in Europe in the sixteenth century.

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\*Smith, *Bulletin of the American Mathematical Society*, vol. 13, page 394.